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# ASYMPTOTIC PROPERTY FOR BAYESIAN SPECIAL TYPE DOUBLE SAMPLING PLAN

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## ABSTRACT

Sampling plans are usually determined by requiring that the Operating Characteristic curve has certain properties and/or by minimizing the average regret. In that way it also becomes possible to study the efficiency of alternative plans and robustness of the solution to changes in the assumption. In this paper the minimum average regret function has been derived for Bayesian Special Type Double Sampling.

**KEYWORDS**: Average Cost, Gamma Distribution, Fixed Second Sample, Minimum Regret

## INTRODUCTION

The best sampling plan is defined as the one within the class minimizing the average costs. The problem is to strike the right balance between manufacturing costs or market price and costs of the inspection procedure including costs resulting from wrong decisions in the case of sampling inspection.

Asymptotic expansions are found under the assumptions that $N \to \infty$, which usually implies that $n \to \infty$ and that $n/N \to 0$. The main advantage of an asymptotic expansion is that it leads to an explicit solution instead of the original implicit one. It clearly shows how the sampling plan depends on the parameters specified, such as quality levels, cost parameters and the parameters of the prior distribution. An asymptotic expansion may be used as starting point for developing approximations which are also valid for small values of N and n.

Hill and Davis (1967) have given the generalized asymptotic expansions of Cornish-Fisher Type. Hald and Keiding (1969) have given the asymptotic properties of Bayesian Decision Rules for Two Terminal Decisions and Multiple Sampling. Hald (1981) has given the asymptotic properties for Bayesian Double Sampling Plan. Pandey (1986) has given the asymptotic solution to Bayesian Three Decision Plan by attributes.

Govindaraju (1984) has developed the Special Type Double sampling plan. In this plan, acceptance is not allowed in the first stage of sampling itself. Although this plan is valid under general conditions for application of attributes sampling inspection, this will be especially useful to product characteristics involving costly or destructive testing, since the acceptance criteria are 0 and 1 for first stage and second stage respectively.

Operating procedure

- Draw a sample of size $n_1$ and observe the number of defectives $d_1$. If $d_1 \leq 1$, reject the lot.
- If $d_1 = 0$, draw a second random sample of size $n_2$ and observe the number of defectives $d_2$. If $d_2 \leq 1$, accept the lot. If $d_2 \geq 2$, reject the lot.

The OC function of the plan is given by

$$P_a = e^{-n}1^p(e^{-n}2^p + n_2pe^{-n}2^p)$$

$$= e^{-np} + n_2pe^{-np}, n = n_1 + n_2$$

$$= P_0P_1 \quad (1)$$

Based on the past history of inspection, it is observed that p follows gamma distribution with density function.

$$w(p) = e^{-p}p^{s-1}t^s/\Gamma(s) \quad (2)$$

The average probability of acceptance is given by

$$\bar{P} = \int P_a\, w(p)dp = S^s/(S+n\mu)^s + n_2\,\mu s^{s+1}/(S+n\mu)^{s+1}$$

Latha and Rajeswari(2012) have given the average regret function for Bayesian Special Type Double Sampling Plan ,

$$R(n_1,n_2,N) = n_1d_s + (N-n_1)d_r + (N-n_1-n_2)\int_0^\infty (p-P_r)P_0P_1dw(p)$$
$$+n_2\int_0^\infty \delta(p)P_0dw(p) \quad (3)$$

Here, we consider the case of destructive inspection, fixed second sample size and continuous prior distribution. Rewriting equation (3),

$$R = n_1d_s + (N-n_1)\int_{-\infty}^{\theta_r}(\theta_r-\theta)w(\theta)d\theta + (N-n_1-n_2)\int_0^\infty(\theta-\theta_r)P_0^{(1)}(\theta)P_1^{(1)}(\theta)w(\theta)d\theta$$
$$+n_2\int_0^\infty \delta(\theta)P_0^{(1)}(\theta)w(\theta)d\theta$$

Where $P_0^{(1)}(\theta) = \Phi\left\{(\theta-h_r)n_1^{1/2}/\sigma(\theta)\right\}, P_1^{(1)}(\theta) = \Phi\left\{(h_a-\theta)n_1^{1/2}/\sigma(\theta)\right\}$.

Changing the variable of integration from θ to $u = (\theta-\theta_r)n_1^{1/2}/\sigma$ and expanding $\sigma(\theta)$ and $w(\theta)$ in Taylor's series around $\theta_r$ we get,

$$R = n_1d_s + 2w\sigma^2n_1^{-1}N\int_0^\infty u\left\{1+O\left(un_1^{1/2}\right)\right\}du$$
$$+2w\sigma^2n_1^{-1}(N-n_2)\int_0^\infty\{u\Phi(\mathrm{z-u}) - \Phi(\mathrm{-z-u})\}\left\{1+O\left(un_1^{1/2}\right)\right\}du$$
$$+\mathrm{N}\sigma^2n_2^{1/2}\int_0^\infty \delta(\theta)\{\Phi(\mathrm{z-u})\}\left\{1+O\left(un_1^{1/2}\right)\right\}du \quad (4)$$

$N-n_1$ is replaced by N since $n_1$ is at most of order $N^{1/2}$. The problem is to determine the values of $n_1$,$n_2$ and z minimizing R. When $n\to\infty$ , the minR is obtained for $z \propto (\ln N)^{1/2}$. The posterior distribution of θ becomes $w_1(\theta)\sim p$. Considering the regret value for first sample and multiplying it with the continuation probability we get,

$$R \sim n_1d_s + 2w\sigma^2n_1^{-1}Nm_2(z) + 2w\sigma^2n_1^{-1}(N-n_2)m_1(z) + w\sigma^2n_2^{1/2}(\delta_r\sigma zN)^{1/2} \quad (5)$$

Setting the derivatives with respect to $n_1$,$n_2$ and z equal to zero, we get,

$$d_s \sim w\sigma^2 n_1^{-2} N m_2(z) + 2w\sigma^2 n_1^{-2}(N - n_2)m_1(z) \tag{6}$$

$$\frac{1}{2} w\sigma^2 n_2^{-1/2}(\delta_r \sigma z N)^{1/2} \sim 2w\sigma^2 n_1^{-1} m_1(z) \tag{7}$$

$$w\sigma^2 n_1^{-1}(N - n_2)m_0(z) \sim w\sigma^2 n_1^{-1} N m_1(z) + \frac{1}{2} w\sigma^2 n_2^{-1/2} z^{-1}(\delta_r \sigma z N)^{1/2} \tag{8}$$

Multiplying equation (7) by $\frac{1}{2} z n_1^{-1} N$, inserting into equation (8) and using that

$m_2(z) + z m_1(z) = m_0(z)$, we get,

$$n_1^2 \sim w\sigma^2 m_0(z)\, N / d_s \tag{9}$$

Solving equation (7) for $n_1$, we find, $n_1^{-1} \sim \delta_r^{-1/2} \sigma^{-1/2} z^{-1/2} m_1(z)$ (10)

Using, $m_0(z) = \phi(z) z^{-1}\left(1 + O(z^{-2})\right)$, $m_1(z) = \phi(z) z^{-2}\left(1 + O(z^{-2})\right)$ and

$m_2(z) = \phi(z) z^{-3}\left(1 + O(z^{-2})\right)$, we get $\left(\phi(z)\right)^4 z^{-5} \sim 2^2 d_s^{-1} \delta_r^2 w\sigma^4 N^{1/2}$ (11)

Taking logarithm, we get, $z \sim (2 \ln N)^{1/2}$ (12)

Having found z we may find $n_1$ and $n_2$. To find min R, we note that

$m_2(z) \sim 2\phi(z) z^{-3} \sim 2 m_0(z) z^{-2}$ so that the second term of equation(5) becomes,

$$w\sigma^2 n_1^{-1} N m_0(z)(2z^{-2}) \sim 2 d_s n_1 z^{-2} \tag{13}$$

Since, $m_1(z) \sim 2\phi(z) z^{-2} \sim 2 m_0(z) z^{-1}$ the third term of equation(5) becomes,

$$w\sigma^2 n_1^{-1}(N - n_2) m_0(z) z^{-1} \sim 4 d_s n_2 z \tag{14}$$

To evaluate fourth term of equation(5), we use equation(8) to eliminate $(\delta_r \sigma z N)^{1/2}$ and noting that, $z m_1(z) \sim m_0(z)$

We get, $3w\sigma^2 n_1^{-1}(N - n_2) m_1(z) z^{-1} \sim 3 d_s n_2 z$ (15)

From the sum of the three terms,

$$\min R \sim n_1 d_s(1 + 2z^{-2}) + 7 d_s n_2 z \sim 3 n_1 d_s + 7 n_2 d_s$$

## CONCLUSIONS

The formula obtained here is useful to obtain fairly accurate plans which may be very useful for the engineers to maintain the quality in industry. It is observed that the contribution of the first stage is one-third of the total minimum regret and the contribution from the second stage is one-seventh in case of fixed second sample size.

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